How Many Fund Managers Does a Fund of Funds Need?

By

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Abstract: We investigate the appropriate number of hedge funds to be held in a fund-of - funds portfolio with the purpose of meeting a typical benchmark mandated by institutional investors. The technique of random sampling from a universe of managers in the Credit Suisse/Tremont hedge fund index is used to generate portfolios. We use naive diversification as well as strategy diversification methods, where the latter simulates a diversified fund-of-funds portfolio. We conclude that forty managers is sufficient to beat the chosen benchmark.

Keywords: fund-of-funds, portfolio of hedge funds, optimal number of managers, naïve diversification, strategy diversification

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1. Introduction

Over the past several years investments in the hedge fund asset class have increased dramatically. According to Hedge Fund Research's 2007 year end report, the total assets managed by hedge funds have increased from $ 490 billion in the year 2000 to $ 1,868 billion in 2007. Interestingly, the total assets managed by fund-of-funds ($ 798 billion) now account for about 42% of the total assets managed by hedge funds. The assets managed by fund-of-funds have increased nine fold since 2000, when it represented only 16% of the total hedge fund assets. In a recent report published by *Preqin Hedg*e[2], 60% of the capital invested in fund-of-funds come from institutional investors. The reason for the increased popularity of hedge funds among institutional investors is their ability to generate high risk adjusted returns, while at the same time having a low correlation to traditional asset classes such as stocks and bonds. Investing via fund-of-funds give investors exposure to a diversified set of hedge fund managers following different strategies (like long/short equity, convertible arbitrage etc.). A majority of fund-of-funds have a diversified approach whereby investment is made in multiple hedge fund strategies, although there are fund-of-funds which are strategy specific.

Selecting managers in the portfolio is obviously very critical to the performance of the portfolio, but at the same time it is important to decide upon the number of managers needed in the portfolio. Since Markowitz (1952), portfolio diversification has been a traditional way of reducing risk. In particular, the optimal number of securities in a portfolio has been widely debated, more recently for example by Statman (1987) for stocks and by L'habitant (2002) and Amo (2007) for hedge funds. Increasing the number of securities (in our case hedge fund managers) in a portfolio results in lower volatility if the securities have low correlation with each other, but the price of lower volatility is usually lower return. Here we investigate the appropriate number of hedge funds in a portfolio from the perspective of an institutional diversified fund-of-funds investment management mandate.

Typically the benchmark given to an absolute return mandate fund-of-funds by its investors ranges from T-Bill to T-Bill + 5%, with an incentive fee for out performance. In addition, many fund-of-funds investors are involved in a portable alpha swap, which obligates them to pay the "risk free" rate[3], so it is important that they at least earn T-Bill returns. A fund-of-funds management firm is more averse to the risk of underperforming the benchmark than it is enthusiastic about the chance of obtaining higher returns. We consider a hypothetical benchmark of T-Bill + 2.5%, which falls inside the range of typical benchmarks used.

The risk of underperforming the benchmark is the main risk we consider in this paper. With this objective in mind, we study the risk of underperforming the benchmark as a function of the number of managers in the portfolio. In order to study the risk return

[2] http://hedge.preqin.com/

[3] An example of a portable alpha trade by an institutional investor is an initial investment in a fund-of-funds manager (alpha source), and a simultaneous entry into a total return swap contract of a desired market index (beta source) whereby the investor agrees to periodically pay returns of T-Bill + spread in exchange for index returns. This means that the responsibility of achieving T-Bill returns falls on the fund-of-funds manager.

characteristics of portfolios of different sizes, we randomly generate portfolios from the selected universe via simulation. The universe of hedge funds that we use is constructed from the managers in the Credit Suisse/Tremont hedge fund index. The reason for choosing this manager set is because the index rules for inclusion are similar to what an institutional investor would require (as we discuss later).

A portfolio of size *M* is simulated by randomly drawing *M* funds from the universe. Hundreds of such portfolios are created for a selected size *M*, for the years 2002 to 2006, and the probability of annual performance exceeding the benchmark for the corresponding year is computed. We will show that as *M* increases, the probability of outperforming the benchmark increases, while at the same time the probability of generating higher returns decreases.

We consider two methods of simulation: naïve diversification and strategy diversification. Naïve diversification is implemented via simple random sampling whereby the strategy type of a randomly selected fund is ignored when making the selection. To simulate performance of a typical fund-of-funds portfolio that is diversified across different strategies, we use the method of strategy diversification implemented via stratified random sampling whereby we constrain the number of funds to be drawn from each strategy. We compare the performance of portfolios generated from the two methods of simulation.

We conclude that a portfolio of approximately forty funds is appropriate for outperforming the benchmark with a high confidence level.

2. Benchmark

As mentioned above, the hypothetical benchmark that we consider is T-Bill + 2.5% , which is shown in Figure 1 for different years. Note that the returns reported in the remainder of the paper are all in excess of the benchmark returns.

Figure 1: Benchmark for different years

		2002	2003	2004	2005	2006
T-Bill*		1.8%	1.2%	1.3%	3.1%	4.9%
Benchmark	**(T-Bill + 2.5%)**	4.3%	3.7%	3.8%	5.6%	7.4%

* Annual 3-month T-Bill returns, Source: Merrill Lynch

3. Universe

We use the managers in the Credit Suisse/Tremont hedge fund index as our initial universe. The reason for selecting the CS/Tremont index as opposed to other indices like HFR (Hedge Fund Research) index is that the former has stricter inclusion requirements (for e.g., a fund needs to have AUM of at least $50 million to be considered to be part of the index and a fund needs to have at least one year of track record)[4], similar to the requirements of a typical institutional investor.

[4] The methodology of index rules is available on the following website: http://www.hedgeindex.com

The universe is created by identifying the funds in the CS/Tremont index from the Lipper TASS and HFR hedge fund databases. We only include funds with full five year track records from 2002 to 2006 in the universe (totaling 254)[5].

Figure 2 shows the strategy constitution of the universe. As can be seen from the table, the universe is skewed by the number of long-short managers. Figure 3(a) tabulates the average excess returns of the funds in the universe for different years, while Fig. 3(b) tabulates the average excess returns of the funds strategy by strategy. If we choose to invest in all 254 funds equally we would get the returns shown in Fig. 3(a) and since the excess returns (over the benchmark) for all years are positive, we would attain our objective of beating the benchmark. Alternatively, if we want to diversify across different strategies, we can invest equally in all funds within each strategy, and invest equal dollar amounts per strategy, and would get returns shown in Fig. 3(b), and this way too we would be able to beat the benchmark. But the question is whether we can invest in fewer funds, thus offering higher return potential and still be able to meet the benchmark with a high degree of confidence. We address this question by randomly creating portfolios of different sizes, and present the methodology and results in the following sections.

Figure 2: Universe composition

	Strategy	Number of funds	Percentage
1	Convertible Arbitrage	5	2%
2	Event	24	9%
3	Equity Market Neutral	17	7%
4	Fixed Income	18	7%
5	Global Macro	8	3%
6	Long Short	107	42%
7	Managed Futures	24	9%
8	Multi Strategy	17	7%
9	Short Bias	5	2%
10	Emerging Market	29	11%
	Total	254	

[5] There is a survivorship bias introduced here because we consider only the funds that have entire five year history. However, as part of our study we look at the return and risk characteristics of portfolios with zero turnover from year 2002 to 2006, in which case this requirement (of 5 year track record) of inclusion in the universe becomes important.

Figure 3 (a)-(b): Universe average returns for different years

(a) Average fund excess return

2002	2003	2004	2005	2006
1.3%	18.4%	6.2%	8.1%	4.5%

(b) Average strategy excess return

	Strategy Returns *	2002	2003	2004	2005	2006
1	Convertible Arbitrage	15.3%	19.7%	0.3%	-7.6%	8.3%
2	Event	-2.6%	23.1%	14.6%	6.6%	11.9%
3	Equity Market Neutral	0.3%	0.1%	-0.8%	0.5%	1.4%
4	Fixed Income	10.4%	5.6%	2.5%	-0.9%	0.6%
5	Global Macro	1.8%	21.2%	1.7%	1.7%	-6.3%
6	Long Short	-5.4%	20.6%	8.9%	9.7%	4.3%
7	Managed Futures	13.7%	9.1%	-1.4%	0.2%	-0.5%
8	Multi Strategy	-2.7%	26.8%	-0.6%	19.7%	3.1%
9	Short Bias	25.0%	-30.2%	-9.0%	5.5%	-12.6%
10	Emerging Market	10.2%	35.5%	10.6%	18.6%	13.7%
	Average	6.6%	13.1%	2.7%	5.4%	2.4%

* Funds within each strategy are equally weighted to obtain the strategy returns

4. Methodology of random sampling

We study the impact of varying the number of funds (M) in a portfolio by the technique of random sampling. We use simple and stratified random sampling to create portfolios as described below.

4.1 Naïve Diversification

We randomly select M funds from the universe and equally weight them to create a portfolio. In this method the strategy type or style of the managers selected is ignored while making the selection, so we could end up with a portfolio of managers of the same style. Although one can select M–fund portfolios from a population of size N in N-choose-M possible ways ($^{N}C_{M}$), we restrict the sample size to be 1,000. Our construction algorithm proceeds for each year between 2002 and 2006 as follows:

1. Draw 10 (M = 10) funds from the universe, and equally weight them to create a portfolio. Create 1,000 such randomly generated portfolios.

2. Select the year 2002
3. Calculate the annual return and record whether it is above or below the benchmark for that year for each of the 1,000 portfolios.
4. Determine the average, the 5th and 95th percentile of 1,000 portfolios excess returns.
5. Repeat steps 3 and 4 for each of the remaining years from 2003 to 2006.
6. Repeat steps 1 to 5 for different values of *M*.

4.2 Strategy Diversification

As pointed out in section 3, the universe is skewed by long-short managers (representing 42% of the total funds in the universe), and if we construct equal-weighted portfolios by simple random sampling, most of the portfolio performance will be dominated by long/short managers. A typical absolute return focused fund-of-funds would diversify its investment across different strategies. In order to simulate diversified portfolios we use the technique of stratified sampling.

The construction algorithm is exactly the same as the one described for simple random sampling except we require an equal number of funds to be drawn from each of the ten strategies. We construct portfolios of size *M* (in multiples of 10) and draw *M/10* funds from each strategy. Since many of the strategies have only five funds, we choose a maximum *M* equal to fifty. By imposing a constraint of equal investment in each strategy, we assume a simple strategy allocation of equal weighting.

5. Simulation Results

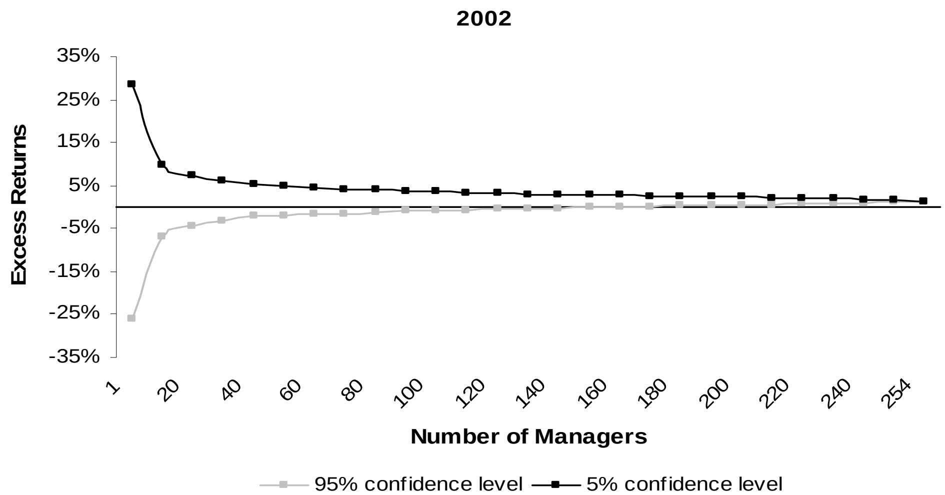
In this section we discuss the results of the simulations. Using the procedures described in the previous section, we construct portfolios of different values of *M*. Figures 4 and 5 show returns at 95% and 5% confidence levels as a function of portfolio size for years 2002, 2004 and 2006 (charts for the three years out of five years are shown for illustration). Figure 6 tabulates the excess returns over the benchmark at a 95% confidence level for portfolios with varying *M*. Figure 7 shows the scatter plot excess returns for portfolios of size forty for the year 2006.

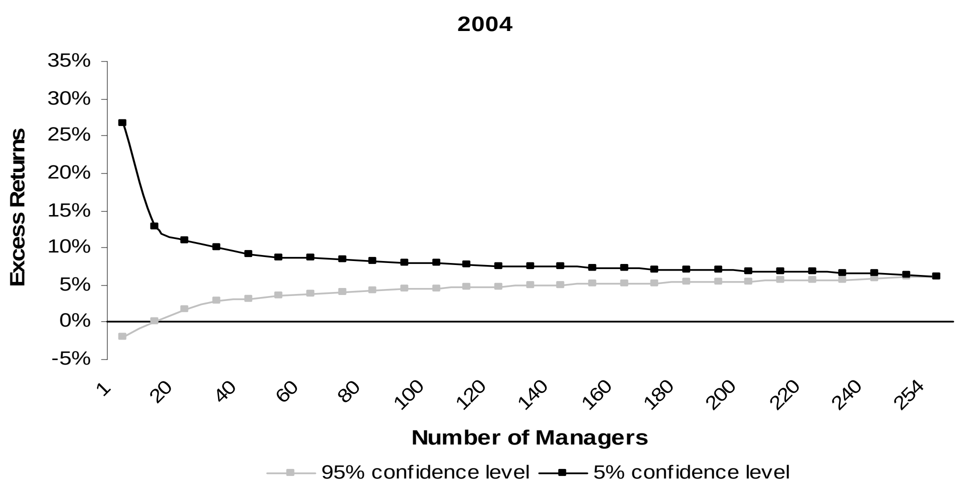
95% (5%) confidence level is the same as the 5th (95th) percentile point. A randomly generated portfolio of same size *M*, has a 95% probability of exceeding the 95% confidence level return and a 5% probability of exceeding the 5% confidence level return.

5.1 Naïve Diversification versus Strategy Diversification

Figures 4 and 5 show the 5% and 95% confidence levels of excess returns as a function of *M* using naïve diversification and strategy diversification methods respectively. For a given *M*, if the 95% confidence level line is above the x-axis (value becomes greater than zero) then it means that a randomly generated portfolio of the same size can beat the benchmark with 95% probability. So for example in Figure 5 for the year 2004, a randomly selected portfolio of size 30 has a 95% chance of beating the benchmark. For both the methods, the 95% confidence level line increases with increasing *M*. What it means is that as *M* increases, it becomes easier and easier to beat the benchmark.

Figure 4: 95% and 5% confidence levels as a function of size using Naïve Diversification





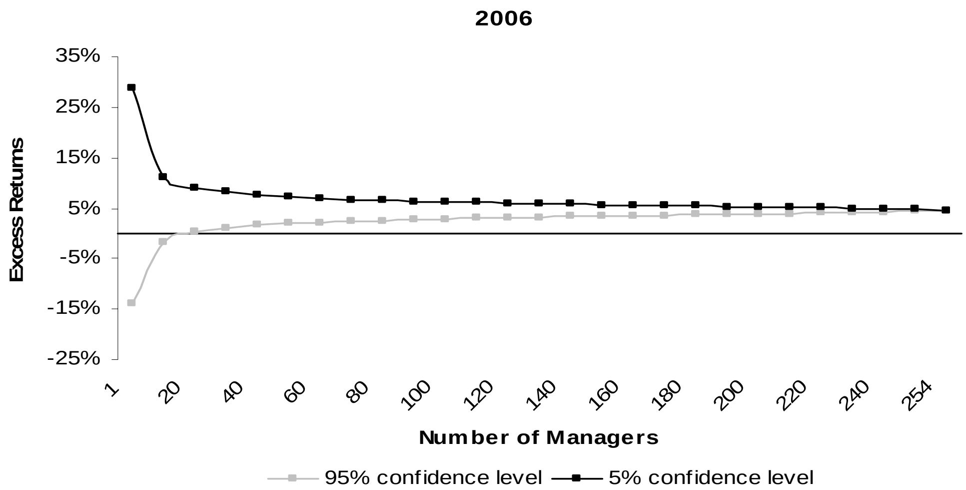
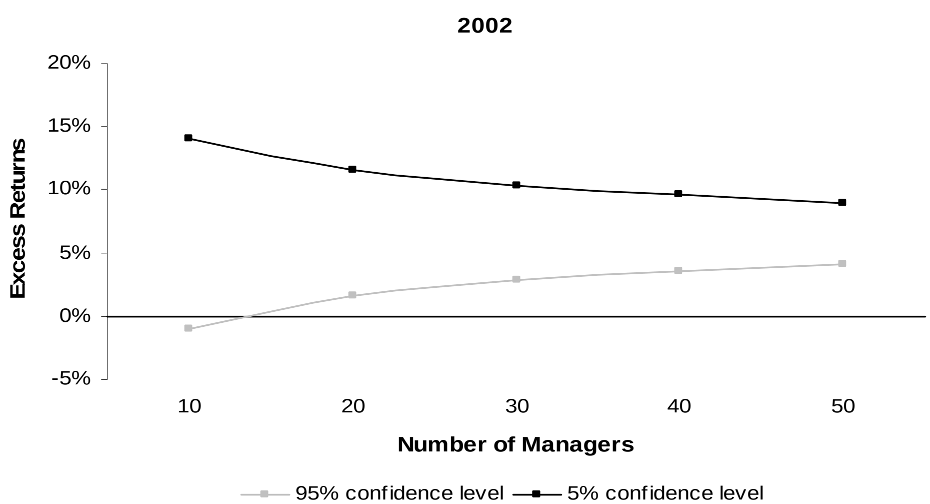
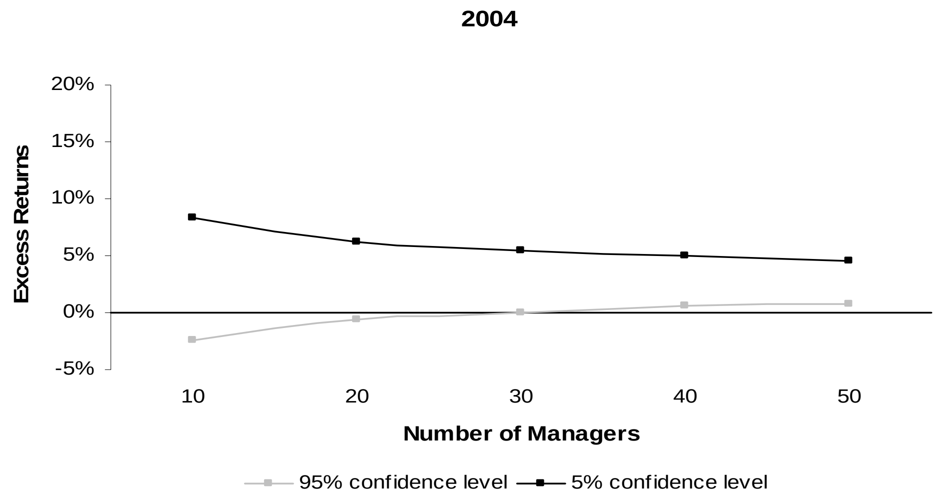
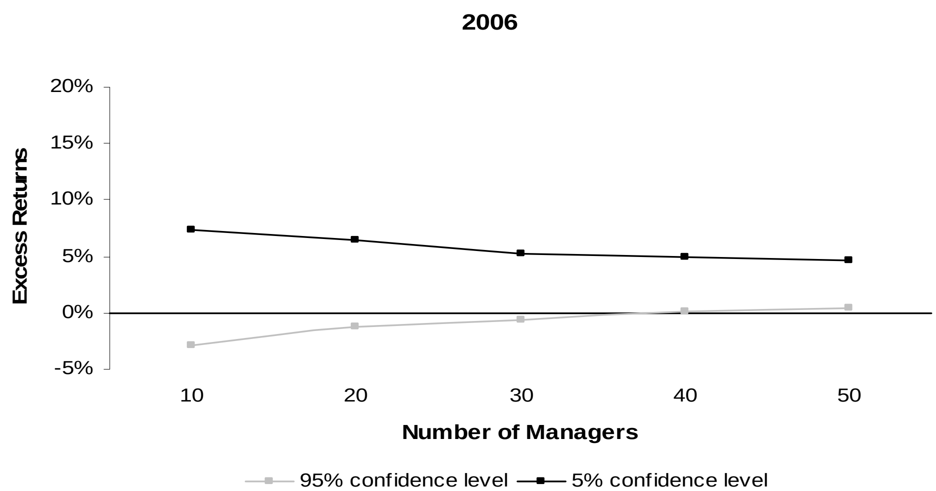


Figure 5: 95% and 5% confidence levels as a function of size using Strategy Diversification







The 95% confidence level returns are also tabulated in Figure 6 as a function of *M* for all the years. As can be seen from Figure 6(a), using the method of naïve diversification, a portfolio of size twenty is able to beat the benchmark (corresponding to positive values) with 95% probability for all years except for the 2002 (a portfolio size of more than 150 is required to beat the benchmark as seen from Figure 4). The reason for poor performance is that 2002 was a bad year for long short equity managers (see Figure 3(b)), and since they occupy the majority of universe (42%) this affects the performance of portfolios created by naive diversification. On the other hand, portfolios with size forty generated by the method of strategy diversification are able to beat the benchmark (indicated by positive values in Figure 6(b)) in each of the five years with 95% probability. The portfolio created by the method of strategy diversification equally weights different strategies, and is not affected by extreme under performance or over performance by any one strategy.

Figure 6: Excess returns at 95% confidence level for portfolios with different sizes

(a) Naïve Diversification

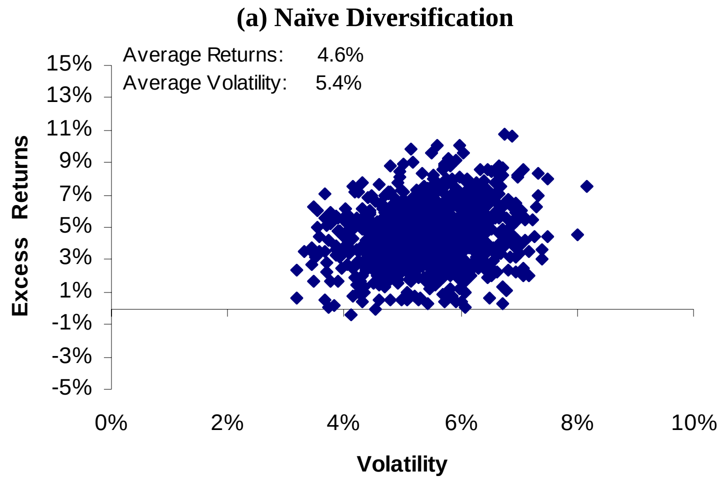
M	2002	2003	2004	2005	2006
10	-7.1%	7.1%	0.0%	0.1%	-1.9%
20	-4.3%	9.9%	1.7%	2.3%	0.2%
30	-3.4%	11.9%	2.8%	3.3%	1.0%
40	-2.0%	12.5%	3.1%	3.6%	1.7%
50	-2.0%	12.9%	3.5%	3.9%	1.9%
60	-1.8%	13.6%	3.8%	3.8%	2.1%

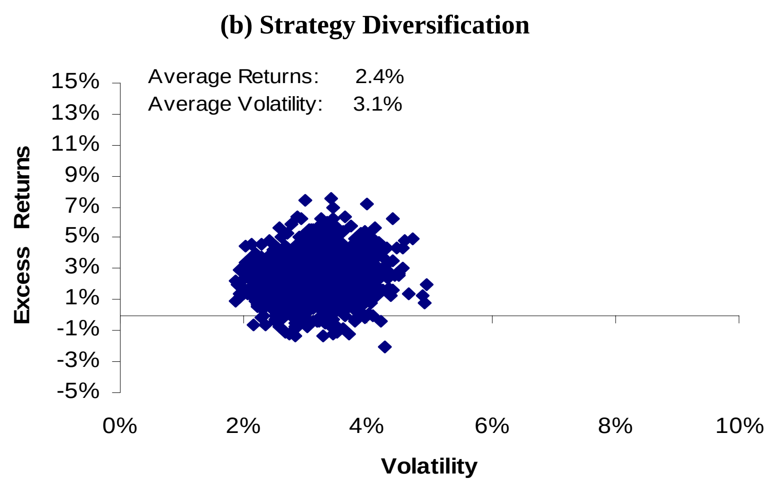
(b) Strategy Diversification

M	2002	2003	2004	2005	2006
10	-0.9%	4.3%	-2.4%	-1.6%	-2.9%
20	1.6%	6.6%	-0.7%	-0.1%	-1.3%
30	2.8%	7.7%	0.0%	0.7%	-0.6%
40	3.6%	8.6%	0.6%	1.4%	0.1%
50	4.1%	9.1%	0.8%	1.7%	0.4%

Figure 7 shows the scatter plot and histogram of portfolios of size forty for the year 2006. As seen from the scatter plot, the average volatility of portfolios using strategy diversification method is much lower (3.1%) then the average volatility using naïve diversification random sampling (5.4%). The reason is strategy diversification offers more diversification (by equally weighting all the strategies) than naïve diversification and thus results in a lower portfolio volatility on average.

Figure 7 : Scatter plots of portfolios of size M = 40 for the year 2006

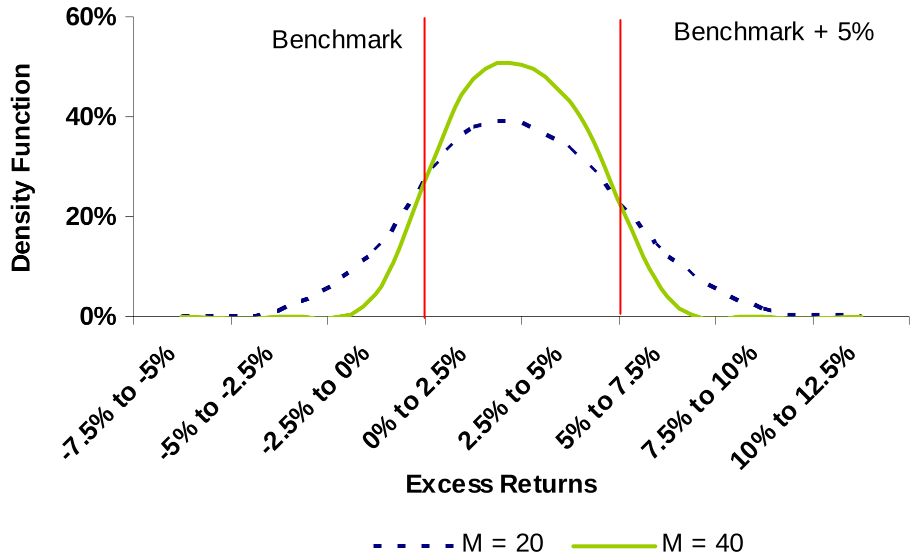




5.2 The limitation of diversification

Figure 8 shows the probability density function of portfolios of size M = 20 and 40 for the 2006 using strategy diversification method. As can be seen from the figure, portfolios of size M = 20 have a higher range of returns as compared to M = 40 sized portfolios. Portfolios of size forty have higher probability of outperforming the benchmark (measured by the area under the curve to the right of the benchmark line) as compared to portfolios of size M = 20, but at the same time they have lower probability of exceeding benchmark + 5% returns (as measured by the area under the curve to the right of the benchmark + 5% line). A similar result can be seen from Figure 5 where the range of returns decreases with increasing *M*, which means that increasing the value of *M* limits the maximum returns that a portfolio can achieve. Clearly the downside protection offered against the benchmark by a high value of *M* comes with the cost of giving up higher returns that a lower *M* can offer.

Figure 8: Portfolio returns density function for 2006



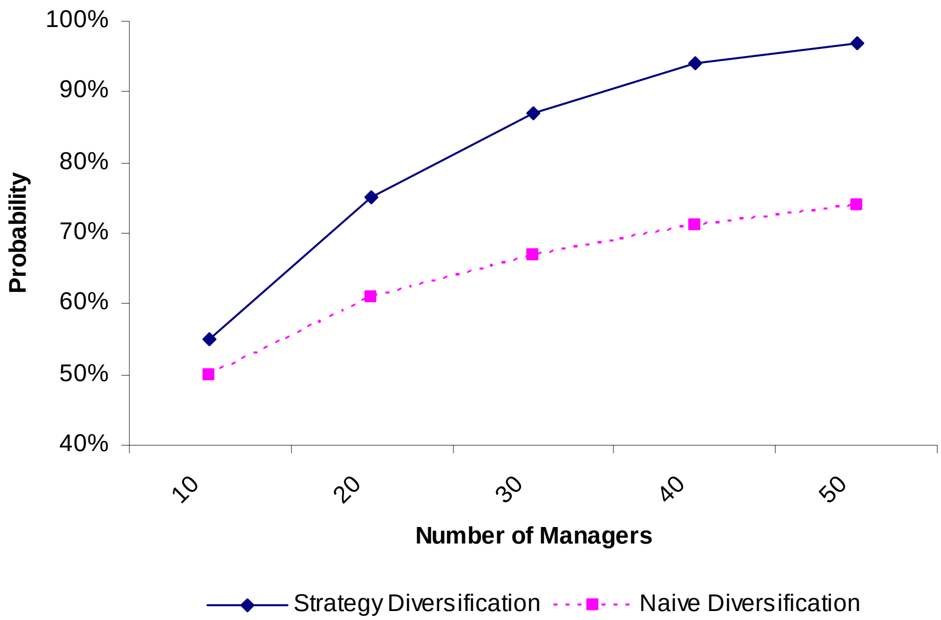
5.3 Zero turn over portfolios – an extreme case

So far we have focused on how different simulated portfolios perform relative to the benchmark on a year to year basis. Our analysis does not take into account the case when a portfolio beats the benchmark in one year but underperforms the benchmark in another year. In order to address the path dependence of performance of portfolios, we consider zero turnover portfolios. We analyze the risk and return performance of a zero turnover portfolio by taking a portfolio at the beginning of 2002 and holding its allocations constant for the entire period and comparing the returns with the benchmark for each year[6]. The probability of portfolios beating the respective benchmarks in all the years is given in Figure 9 as a function of portfolio size. As can be seen from the figure, the probability of beating the benchmark using strategy diversification method exceeds that of portfolios generated using naïve diversification for all values of *M*. Also, portfolios of sizes forty and fifty using strategy diversification method have a very high probability (94% and 97% respectively) of beating the benchmark.

[6] The probability of successful consecutive outcomes of an event would be less than probability of individual successes. For example, if we consider two tosses of a fair coin, the probability of getting a heads on the first toss (or the second toss) is 0.5. However, the probability of getting two heads consecutively is 0.25 (0.5x0.5)

In reality a fund-of-funds won't have zero turn over for a period of five years. A fund-of-funds management firm can enhance the performance of these constantly held portfolios, if it can correctly predict and move capital from underperforming managers (strategies) to outperforming managers (strategies).

Figure 9: Probability of a zero turn over portfolio beating the benchmark in each of the consecutive years as a function of *M*



Based on the results of individual year by year performance of portfolios and performance of zero turnover portfolios across all the years, we find that given our objective of beating the benchmark with a high confidence, forty managers is an appropriate number to have in a portfolio, assuming a diversified fund-of-funds with an absolute return mandate.

6. Conclusion

Although portfolio construction is not typically based on the random selection of funds, our study of randomly generated portfolios illustrates the range of possible returns as a function of the number of managers. We believe portfolios generated by strategy diversification are representative of a typical diversified fund-of-funds portfolio. For the universe, benchmark and the time period selected in our study, we conclude that a diversified portfolio of approximately size forty is optimal for a fund-of- funds portfolio. However, we note that the following factors can influence the number of managers required:

1. **Investment objective:** In this paper we use a hypothetical benchmark (T-Bill +2.5%) catering to the needs of an institutional investor. All else equal, increasing (decreasing) the required benchmark or the confidence level at which the return exceeds the benchmark will require a higher (lower) number of managers to be included in the portfolio.

2. **Strategy allocation:** We used a very simple strategy allocation (with all strategies equally weighted) to simulate a typical fund-of-funds portfolio. A strategy allocation based on correctly forecasting underperforming/outperforming strategies and allocating accordingly will achieve better performance than by equally weighting all strategies. All else equal, a better performing strategy allocation would necessitate a lower number of managers to be included in the portfolio.

3. **Manager selection:** Manager selection has a major impact on portfolio performance as illustrated in the study by Reddy (2007). If a fund-of-funds manager is able to predict and overweight outperforming managers, better performance can be achieved. Superior manager selection would imply a portfolio with a lower number of managers could still meet the benchmark.

The topic of optimal number of hedge funds required in a portfolio has been studied over the past several years. The optimal number of funds suggested by other studies range from as low as 5 to 20 funds. L'habitant (2002) concludes no more than 5 to 10 funds are required to diversify the portfolio risk away, Amo (2007) suggests that the marginal benefit of adding a new fund decreases beyond 6 funds. Amin (2002) concludes that more than 15 funds is not necessary to have in a portfolio.

The reason why our research shows a different number of optimal managers can be attributed to one or all of the following factors: the starting universe, risk criteria and simulation methodology. We use the manager set represented in CS/Tremont hedge index as our starting universe (which we believe is representative of funds that institutional investors would invest in), while other research mainly use larger databases (in which a majority of the funds have AUM less than $50 million, typically not considered investable by institutional investors). The main risk we consider to determine optimal number of managers is that of underperforming a benchmark typically mandated by institutional investors, while other research studies use statistical return parameters such as volatility, correlation, drawdown etc. Almost all research use only naïve diversification to simulate portfolio returns. Along with naïve diversification, we also use strategy diversification which we believe is more representative of a diversified fund-of-funds portfolio.

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